Collisionless damping of nonlinear dust ion acoustic wave due to dust charge fluctuation

Samiran Ghosh,* Tushar K. Chaudhuri, Susmita Sarkar,† Manoranjan Khan,‡ and M. R. Gupta
Centre for Plasma Studies, Faculty of Science, Jadavpur University, Calcutta 7000 032, India

(Received 1 March 2001; revised manuscript received 24 August 2001; published 21 March 2002)

A dissipation mechanism for the damping of the nonlinear dust ion acoustic wave in a collisionless dusty plasma consisting of nonthermal electrons, ions, and variable charge dust grains has been investigated. It is shown that the collisionless damping due to dust charge fluctuation causes the nonlinear dust ion acoustic wave propagation to be described by the damped Korteweg-de Vries equation. Due to the presence of nonthermal electrons, the dust ion acoustic wave admits both positive and negative potential and it suffers less damping than the dust acoustic wave, which admits only negative potential.

DOI: 10.1103/PhysRevE.65.037401 PACS number(s): 52.35.Fp

Both theoretical [1–9] and experimental [10–12] studies of nonlinear features of ''dust acoustic'' [13] and ''dust ion acoustic'' [14] waves have been done recently either assuming fixed charge on the dust grain or by approximating the charging equation by $I_e+I_i\approx 0$, which implies that the dust charge Q_d instantaneously reaches equilibrium value at each space-time point determined by the local electrostatic potential $\phi(x,t)$.

The nonadiabaticity effect of dust charge fluctuation causing generation of dust acoustic and dust ion acoustic shock wave with their propagation described by the Korteweg-de Vries (KdV) Burger equation has recently been studied by Gupta *et al.* [15] and Ghosh *et al.* [16] under the assumption $\omega_{\mathrm{pd}}\tau_{\mathrm{ch}}$ is small but finite, where ω_{pd} is the dust plasma frequency and τ_{ch} is the dust charging frequency. This is an interesting mechanism for the generation of shock waves in a dusty plasma.

In this brief report, the nonlinear propagation of finite but small amplitude dust ion acoustic waves in a collisionless dusty plasma in which electrons are nonthermal has been investigated under the opposite assumption viz. $1/\omega_{\mathrm{pi}}\tau_{\mathrm{ch}}$ is small but finite, where ω_{pi} is the ion plasma frequency. Such a situation may occur in the ionosphere ($\sim$80 KM). It is shown that the propagation is governed by the damped KdV equation. The presence of nonthermal electrons may cause generation of solitary dust ion acoustic wave with negative electrostatic potential. The damping arises due to the dust charge fluctuation under the assumption stated above.

To ensure ease of following the text, the symbols for different physical parameters and the normalizations of the physical variables are listed below.

The normalized space X, time T, velocity V_j $(j=i,d)$, number density fluctuation N_j $(j=i,d)$, the dust charge fluctuation ΔQ, electrostatic potential Φ, and total energy E of electrons are given by $X=x/\lambda_i$; $T=\omega_{\mathrm{pi}}t$; $V_j=v_j/c_i$; $n_j=n_{j0}N_j$, the dust charge $Q_d=z_de(-1+\Delta Q)$; $\Phi=e\phi/T_e$, $E=(\frac{1}{2}m_ev_e^2-e\phi)/T_e$, where $\lambda_i=\sqrt{\epsilon_0T_e/n_{i0}e^2}$ is the ion Debye length, $\omega_{\mathrm{pi}}=\sqrt{n_{i0}e^2/\epsilon_0m_i}$ the ion plasma frequency, and $c_i=\sqrt{T_e/m_i}$ is the ion acoustic speed.

The electron distribution function with a population of fast particles is assumed to be given by Cairns *et al.* [17] $f(E)=n_{e0}(1+aE^2)e^{-E}/(1+3a)$, so that the electron number density is $n_e=n_{e0}[1-b(\Phi-\Phi^2)]\exp(\Phi)$. Here, $b=4a/(1+3a)$, E is the normalized electron energy as stated above, and a is the parameter determining the number of fast electrons.

The other parameters are the following: r_0 is the grain radius, $-z_de$ is the equilibrium surface charge on the dust grain, σ is the electron ion temperature ratio, i.e., T_i/T_e, $\delta=n_{e0}/n_{i0}$, $z=z_de^2/4\pi\epsilon_0r_0T_e$, $\mu_d=z_dm_i/m_d$, and $\nu_{\mathrm{ch}}=r_0/\sqrt{2\pi}\omega_{\mathrm{pi}}^2/V_{\mathrm{thi}}(1+z+\sigma)$ is the dust charging frequency, where V_{thi} is the ion thermal velocity and $\tau_{\mathrm{ch}}=\nu_{\mathrm{ch}}^{-1}$ is the charging time.

The dynamics of dust ion acoustic waves are governed by the following normalized continuity and momentum equations for the dust fluid and ion fluid together with Poisson's equations [16]:

$$\frac{\partial N_d}{\partial T}+\frac{\partial(N_dV_d)}{\partial X}=0, \tag{1}$$

$$\frac{\partial V_d}{\partial T}+V_d\frac{\partial V_d}{\partial X}=-\mu_d(\Delta Q-1)\frac{\partial\Phi}{\partial X}, \tag{2}$$

$$\frac{\partial N_i}{\partial T}+\frac{\partial(N_iV_i)}{\partial X}=0, \tag{3}$$

$$\frac{\partial V_i}{\partial T}+V_i\frac{\partial V_i}{\partial X}=-\frac{\partial\Phi}{\partial X}-\frac{\sigma}{N_i}\frac{\partial N_i}{\partial X}, \tag{4}$$

$$\frac{\partial^2\Phi}{\partial X^2}=\frac{1}{n_{i0}}[n_e-n_{i0}N_i-z_dn_{d0}N_d(\Delta Q-1)]. \tag{5}$$

In deriving Eq. (5), the equilibrium state charge neutrality condition $n_{i0}=n_{e0}+z_dn_{d0}$ has been used where n_{j0} $(j=e,i,d)$ are the equilibrium number densities and $(-z_de)$ is the equilibrium charge of the dust surface.

The normalized charge variable ΔQ is determined by the following orbital motion limited dust charging equation [18]:

*Also at Netaji Subhas Engineering College, Techno City, Panchpotha, Police Para, Garia, Calcutta, India. Email address: sran@jufs.ernet.in; sran_g@yahoo.com
†Department of Applied Mathematics, University of Calcutta, 92, A.P.C. Road, Calcutta 7000 009, India.
‡Email address: mk@jufs.ernet.in; mkhan_ju@yahoo.com

$$\frac{\partial \Delta Q}{\partial T}+V_d\frac{\partial \Delta Q}{\partial X}=\frac{1}{\omega_{\mathrm{pi}}\tau_{\mathrm{ch}}}\frac{(I_e+I_i)}{\nu_{\mathrm{ch}}z_d e},\tag{6}$$

where I_e and I_i are the electron and ion current, respectively. The normalized expressions for the electron and ion currents for spherical dust grains with radius r_0 are as follows:

$$I_e=-\pi r_0^2 e\sqrt{\frac{8T_e}{\pi m_e}}n_{e0}[1-b(\Phi-\Phi^2)]\exp(\Phi)\times\exp[z(\Delta Q-1)],\tag{7}$$

$$I_i=\pi r_0^2 e\sqrt{\frac{8T_i}{\pi m_i}}n_{i0}N_i\left[\left(1+\frac{z}{\sigma}\right)-\frac{z}{\sigma}\Delta Q\right].\tag{8}$$

In order to study the nonlinear propagation of small-amplitude dust ion acoustic wave by the reductive perturbation technique [19], the independent variables are stretched as

$$\xi=\epsilon^{1/2}(X-\lambda T);\quad \tau=\epsilon^{3/2}T,\tag{9}$$

where λ is the normalized phase velocity of linear dust ion acoustic wave normalized by the ion acoustic speed and ϵ is a small parameter characterizing the strength of the nonlinearity.

The dependent variables N_d, N_i, V_d, V_i, Φ, and ΔQ are expressed as

$$f=f^{(0)}+\epsilon f^{(1)}+\epsilon^2 f^{(2)}+\cdots\tag{10}$$

where $f^{(0)}=1$ for $f=N_d,N_i$, while for other variables $f^{(0)}=0$.

We now turn to the dust charging Eq. (6). On the right-hand side of Eq. (6), the factor $1/\omega_{\mathrm{pi}}\tau_{\mathrm{ch}}=r_0/\sqrt{2\pi}\omega_{\mathrm{pi}}/V_{\mathrm{thi}}\ (1+z+\sigma)\approx r_0/\lambda_i(1+z+\sigma)$. For dusty plasma parameter $z=2.0$ to $z=2.5$ and $\sigma\approx1$, $1/\omega_{\mathrm{pi}}\tau_{\mathrm{ch}}\approx r_0/\lambda_i\approx10^{-6}$ [20] in the ionosphere ($\sim$80 KM). This suggests that to make the nonlinear perturbation of Eqs. (6)–(8) consistent with that of Eqs. (1)–(5), the following scaling is permissible:

$$\frac{1}{\omega_{\mathrm{pi}}\tau_{\mathrm{ch}}}=\nu\epsilon^{3/2}.\tag{11}$$

We introduce the coefficient ν only to locate in the final expression the effect arising from dust charge variation contributed by the left-hand side of Eq. (14). Subsequently, we shall put $\nu=1$.

Introducing Eqs. (9) and (10) into Eqs. (1)–(5) and equating the terms in lowest powers of ϵ, i.e., $O(\epsilon^{3/2})$, and eliminating $V_d^{(1)}=-\mu_d/\lambda\Phi^{(1)}$ and $V_i^{(1)}=\lambda/(\lambda^2-\sigma)\Phi^{(1)}$, we get

$$(N_d^{(1)},N_i^{(1)})=\left(-\frac{\mu_d}{\lambda^2},\frac{1}{\lambda^2-\sigma}\right)\Phi^{(1)}.\tag{12}$$

Substitution of $n_e=n_{e0}[1-b(\Phi-\Phi^2)]\exp(\Phi)$ in Poisson's Eq. (5) yield to order $O(\epsilon^2)$

$$N_i^{(1)}=(1-\delta)(N_d^{(1)}-\Delta Q^{(1)})+\delta(1-b)\Phi^{(1)}.\tag{13}$$

Using the stretching (9), perturbation expansion (10) for all the variables and the scaling (11), we arrive at

$$\left[(-\lambda\epsilon^{1/2}+\epsilon^{3/2}V_d^{(1)})\frac{\partial}{\partial\xi}+\epsilon^{3/2}\frac{\partial}{\partial\tau}\right](\epsilon\Delta Q^{(1)}+\epsilon^2\Delta Q^{(2)})=\nu\epsilon^{3/2}\frac{(I_e+I_i)}{\nu_{\mathrm{ch}}z_d e}.\tag{14}$$

Introducing the expansions for Φ, ΔQ, and N_d in Eqs. (7) and (8) and equating terms $O(\epsilon^{3/2})$, we get

$$\frac{\partial\Delta Q^{(1)}}{\partial\xi}=0\Rightarrow\Delta Q^{(1)}=\Delta Q^{(1)}(\tau)=0,\tag{15}$$

by virtue of the boundary conditions that all perturbations vanish at $X=-\infty(\xi=-\infty)$ for all time scales slow or fast. In obtaining Eq. (15), we have also used the fact that $(I_e+I_i)=0$ in the equilibrium state, which implies that the charge on each dust grain is fixed $(Q_d=-z_d e)$.

The normalized phase velocity λ is obtained by eliminating $N_i^{(1)}$, $N_d^{(1)}$, $\Phi^{(1)}$, and $\Delta Q^{(1)}$ from the homogeneous system of Eqs. (12), (13), and (15).

$$\lambda^2=\lambda_\pm^2=\frac{\lambda_0\pm\sqrt{\lambda_0^2-4\sigma\mu_d\delta(1-b)(1-\delta)}}{2\delta(1-b)},\tag{16}$$

where $\lambda_0=1+\delta\sigma(1-b)+\mu_d(1-\delta)$. Here, λ_+ and λ_- are the phase velocities of the faster wave (usual dust ion acoustic in the limit $\mu_d\to0$) and the slower (usual dust acoustic) wave. Equating terms $O(\epsilon^{3/2})$, we get from Eqs. (1)–(5),

$$\frac{\partial N_d^{(1)}}{\partial\tau}+\frac{\partial(N_d^{(1)}V_d^{(1)})}{\partial\xi}=\lambda\frac{\partial N_d^{(2)}}{\partial\xi}-\frac{\partial V_d^{(2)}}{\partial\xi},\tag{17}$$

$$\frac{\partial V_d^{(1)}}{\partial\tau}+V_d^{(1)}\frac{\partial V_d^{(1)}}{\partial\xi}+\mu_d\Delta Q^{(1)}\frac{\partial\Phi^{(1)}}{\partial\xi}=\lambda\frac{\partial V_d^{(2)}}{\partial\xi}+\mu_d\frac{\partial\Phi^{(2)}}{\partial\xi},\tag{18}$$

$$\frac{\partial N_i^{(1)}}{\partial\tau}+\frac{\partial(N_i^{(1)}V_i^{(1)})}{\partial\xi}=\lambda\frac{\partial N_i^{(2)}}{\partial\xi}-\frac{\partial V_i^{(2)}}{\partial\xi},\tag{19}$$

$$\frac{\partial V_i^{(1)}}{\partial\tau}-\lambda N_i^{(1)}\frac{\partial V_i^{(1)}}{\partial\xi}+V_i^{(1)}\frac{\partial V_i^{(1)}}{\partial\xi}+N_i^{(1)}\frac{\partial\Phi^{(1)}}{\partial\xi}=\lambda\frac{\partial V_i^{(2)}}{\partial\xi}-\frac{\partial\Phi^{(2)}}{\partial\xi}-\sigma\frac{\partial N_i^{(2)}}{\partial\xi},\tag{20}$$

$$\frac{\partial^2\Phi^{(1)}}{\partial\xi^2}=\left[-N_i^{(2)}+(1-\delta)(N_d^{(2)}-\Delta Q^{(2)}-N_d^{(1)}\Delta Q^{(1)})+\delta(1-b)\Phi^{(2)}+\frac{\delta}{2}\Phi^{(1)^2}\right].\tag{21}$$

Once again, substituting the expansions for N_i Φ, and ΔQ from Eq. (10) into right-hand side of Eq. (14), using Eqs. (12) and (15) and finally equating terms $O(\epsilon^{5/2})$, we obtain

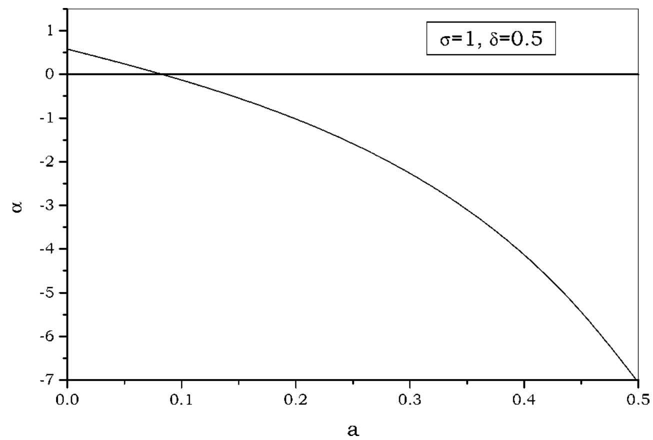

FIG. 1. Variation α [Eq. (25)] with nonthermal parameter a for dust ion acoustic (faster) wave with phase velocity λ_+ [Eq. (16)] and for $\sigma=T_i/T_e=1$, $\mu_d=5\times10^{-5}$, $\delta=n_{e0}/n_{i0}=0.5$, and $z\approx1.97$.

$$\frac{\partial \Delta Q^{(2)}}{\partial \xi}=\frac{\nu}{\lambda}\left(\frac{\sigma+z}{z(1+\sigma+z)}\right)\frac{[(1-b)\lambda^2+b\sigma-(1+\sigma)]}{(\lambda^2-\sigma)}\Phi^{(1)}. \tag{22}$$

Now differentiating Eq. (21) with respect to ξ and using Eq. (15), we get

$$\frac{\partial^3\Phi^{(1)}}{\partial\xi^3}=-\frac{\partial N_i^{(2)}}{\partial\xi}+(1-\delta)\frac{\partial N_d^{(2)}}{\partial\xi}-(1-\delta)\frac{\partial\Delta Q^{(2)}}{\partial\xi}+\delta(1-b)\frac{\partial\Phi^{(2)}}{\partial\xi}+\delta\Phi^{(1)}\frac{\partial\Phi^{(1)}}{\partial\xi}. \tag{23}$$

Finally, on substituting for $\partial\Delta Q^{(2)}/\partial\xi$ from Eq. (22) and on eliminating $\partial/\partial\xi[(1-\delta)N_d^{(2)}-N_i^{(2)}+\delta(1-b)\Phi^{(2)}]$ by using Eqs. (17) through (20), we arrive at the following damped KdV equation

$$\frac{\partial\Phi^{(1)}}{\partial\tau}+\alpha\Phi^{(1)}\frac{\partial\Phi^{(1)}}{\partial\xi}+\beta\frac{\partial^3\Phi^{(1)}}{\partial\xi^3}+\gamma\Phi^{(1)}=0, \tag{24}$$

where

$$\alpha=\beta\left[\frac{(3\lambda^2-\sigma)}{(\lambda^2-\sigma)^3}-\delta-\frac{3\mu_d^2(1-\delta)}{\lambda^4}\right], \tag{25}$$

$$\gamma=\nu\beta\frac{(\sigma+z)(1-\delta)}{z(1+\sigma+z)}\frac{[(1-b)\lambda^2+b\sigma-(1+\sigma)]}{\lambda(\lambda^2-\sigma)}, \tag{26}$$

$$\beta=\frac{1}{2}\left[\frac{\mu_d(1-\delta)}{\lambda^3}+\frac{\lambda}{(\lambda^2-\sigma)^2}\right]^{-1}. \tag{27}$$

Equation (26) shows that the damping γ disappears when we put $\nu=0$, as it should because $\nu=0$ implies the right-hand side of Eq. (14) $=0$ $[dQ_d/dt=0]$ i.e., there is no dust charge variation [by Eqs. (15) and (22) $\Delta Q^{(1)}=\Delta Q^{(2)}=0\Rightarrow$fixed charge on the dust grain surface]. However, we shall put $\nu=1$ for our numerical calculations. It is also to be noted that $\gamma\approx0$ for $\delta=n_{e0}/n_{i0}\approx1$; this is expected as

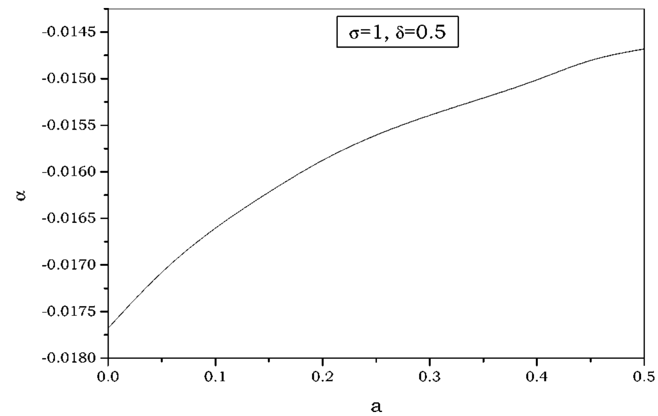

FIG. 2. Variation of α [Eq. (25)] with nonthermal parameter a for dust acoustic (slower) wave with phase velocity λ_- [Eq. (16)]. The other parameters are the same as Fig. 1.

$\delta\approx1$ implies near absence of charged dust grain and consequently no charge fluctuation. Thus, the damping of the dust ion acoustic wave is due to the dust charge variation.

Integrating Eq. (24), using the boundary conditions $N_d^{(1)}$, $N_i^{(1)}\to1$; $V_d^{(1)}$, $V_i^{(1)}$, $\Phi^{(1)}\to0$ as $|X|\to\infty$, we obtain sech^2 time evolution solitary wave form approximate solution as

$$\Phi^{(1)}=\Phi_m^{(1)}(\tau)\mathrm{sech}^2\left[\sqrt{\frac{\alpha\Phi_m^{(1)}(\tau)}{12\beta}}(\xi-V\tau)\right],$$

$$\Phi_m^{(1)}=\Phi_0^{(1)}e^{-\gamma\tau},\quad V=\frac{\alpha\Phi_0^{(1)}}{6}e^{-\gamma\tau}, \tag{28}$$

where $\Phi_m^{(1)}$ is the soliton amplitude. It is to be noted that Eq. (28) represents a positive or negative well potential wave, i.e., $\Phi_m^{(1)}(\tau)>$ *or* <0, according as the coefficient of nonlinear term α in the KdV equation $>$ or $<$ 0.

We summarize the results as follows:

(1) Ion acoustic solitary electrostatic structures involving ion density depletion have been observed in the ionosphere by Freja satellite [21] and motivated by this observation

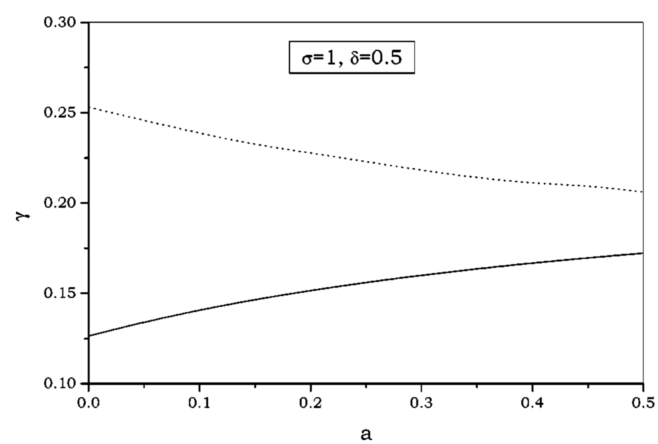

FIG. 3. Variation of γ [Eq. (26)] with nonthermal parameter a. Solid lines for dust ion acoustic (faster) wave and dotted lines for dust acoustic (slower) wave. The other parameters are the same as Fig. 1.

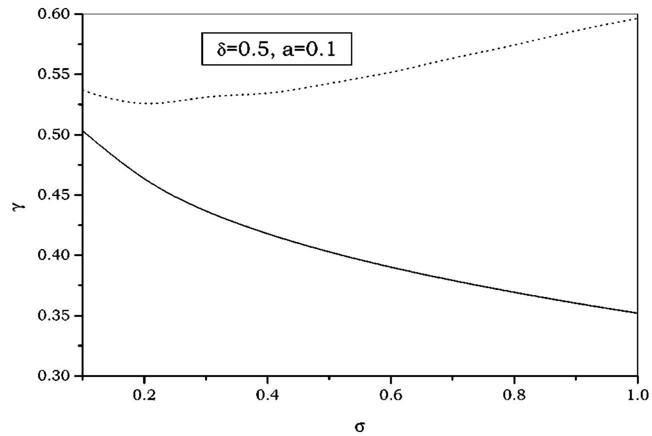

FIG. 4. Variation of γ [Eq. (26)] with $\sigma = T_i/T_e$ for $a = 0.1$. The value of δ and μ_d are the same as Fig. 1. Solid and dotted lines represent the same as in Fig. 3.

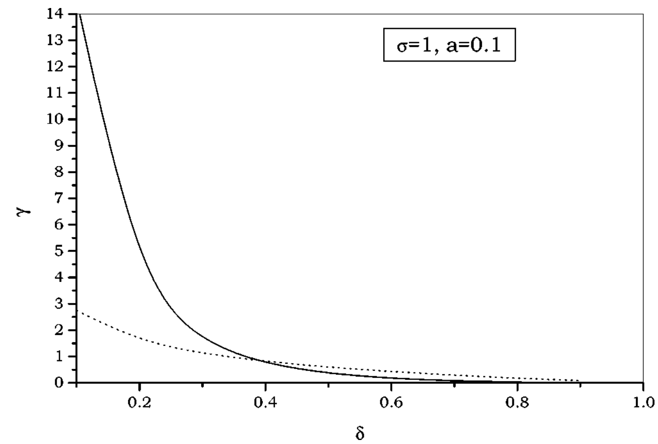

FIG. 5. Variation of γ [Eq. (26)] with $\delta = n_{e0}/n_{i0}$ for $\sigma = T_i/T_e = 1$. The value of a and μ_d are the same as in Fig. 4. Solid and dotted lines represents same as in Fig. 3.

Cairns *et al.* [17] studied the ion solitary wave in presence of fast electrons. Ionospheric plasma ($\sim$80 KM) is dust contaminated and the dust charging time satisfies $1/\omega_{\mathrm{pi}}\tau_{\mathrm{ch}} \ll 1$. Hence, the relevance of the present report, which shows [Fig. 1 and Eq. (28)] that the dust ion acoustic wave ($\lambda = \lambda_+ > \sqrt{\sigma}$) has negative potential for higher values of the electron distribution nonthermality parameter a and so is associated with ion density depletion while for low values of a, the density is enhanced as $\alpha \gtrless 0 \Rightarrow \Phi^{(1)} \gtrless 0 \Rightarrow N_i^{(1)} \gtrless 0$. On the other hand, for dust acoustic wave ($\lambda = \lambda_- < \sqrt{\sigma}$), Fig. 2 together with Eqs. (12) and (28) indicates that $\alpha < 0 \Rightarrow \Phi^{(1)} < 0 \Rightarrow N_i^{(1)} > 0$ (density enhancement) for all values of a lower or higher.

(2) The solitary wave amplitude $\Phi^{(1)} \sim e^{-\gamma\tau}$ so that the attenuation time $t \sim 1/\epsilon^{3/2}\gamma\omega_{\mathrm{pi}}$. Since for ionosphere ($\sim$80 KM), $\epsilon^{3/2} \approx 1/\omega_{\mathrm{pi}}\tau_{\mathrm{ch}} \approx (r_0/\lambda_i)$ [Eq. (11) with $\nu = 1$] is typically $\sim 10^{-6}$, the ion density depletion (or enhancement) will persist for a sufficient period of time and travel a sufficiently large distance $\sim \lambda_+ c_i/\epsilon^{3/2}\gamma\omega_{\mathrm{pi}}$ to be observed.

(3) The damping rate $\gamma(a, \sigma, \delta)$ [Eq. (26)] is a function of a, σ, and δ. Figures 3, 4, and 5 show that variation of γ with respect to a, σ, and δ, respectively, keeping the other two parameters unchanged. Figure 3 shows that for fixed σ and δ, the damping rate γ decreases / increases for waves with $\lambda = \lambda_-/\lambda_+$ as the nonthermality parameter a increases. Both Figs. 3 and 4 show that with increase of a or temperature ratio σ, the slower (dust acoustic) wave always suffers stronger damping than the faster (dust ion acoustic) wave. Figure 5 shows that with increase of δ, γ decreases sharply for dust ion acoustic (faster) wave and $\gamma \approx 0$ for $\delta = n_{e0}/n_{i0} \to 1$. This is to be expected as the latter implies decrease in dust density and hence decrease in dust charge fluctuation induced damping. Similar result though less pronounced also holds for slower wave.

One of the authors (S.G.) would like to thank the Department of Science and Technology (DST), Government of India (Sanction No. SP/12/PC-07/2000) for financial support during this work.

[1] R. K. Varma, P. K. Shukla, and V. Krishan, Phys. Rev. E **47**, 3612 (1993).
[2] F. Melandso, T. Aslaksen, and O. Havnes, Planet. Space Sci. **41**, 321 (1993).
[3] R. Bharuthram and P. K. Shukla, Planet. Space Sci. **40**, 973 (1992).
[4] P. K. Shukla, Phys. Plasmas **7**, 1044 (2000).
[5] N. N. Rao and P. K. Shukla, Planet. Space Sci. **42**, 221 (1990).
[6] F. Verheest, Planet. Space Sci. **40**, 1 (1992).
[7] J. X. Ma and Liu, Phys. Plasmas **4**, 253 (1997).
[8] B. Xie, K. He, and Z. Huang, Phys. Lett. A **247**, 403 (1998).
[9] Samiran Ghosh, S. Sarkar, M. Khan, and M. R. Gupta, Phys. Plasmas **7**, 3594 (2000).
[10] A. Barkan, R. Merlino, and N. D'Angelo, Phys. Plasmas **2**, 3563 (1995).
[11] Y. Nakamura, H. Bailung, and P. K. Shukla, Phys. Rev. Lett. **83**, 1602 (1999).
[12] Q. Z. Luo, N. D'Angelo, and R. L. Merlino, Phys. Plasmas **6**, 3455 (1999).
[13] N. N. Rao, P. K. Shukla, and M. Y. Yu, Planet. Space Sci. **38**, 543 (1990).
[14] P. K. Shukla and V. P. Silin, Phys. Scr. **45**, 508 (1992).
[15] M. R. Gupta, S. Sarkar, Samiran Ghosh, M. Debnath, and M. Khan, Phys. Rev. E **63**, 046406 (2001).
[16] Samiran Ghosh, S. Sarkar, M. Khan, and M. R. Gupta, Phys. Lett. A **274**, 162 (2000).
[17] R. A. Cairns *et al.*, Geophys. Res. Lett. **22**, 2709 (1995).
[18] J. E. Allen, Phys. Scr. **45**, 497 (1992).
[19] H. Washimi and T. Tanuiti, Phys. Rev. Lett. **17**, 996 (1966).
[20] U. de Angelis, Phys. Scr. **45**, 465 (1992).
[21] O. P. Dovner, A. I. Eriksson, R. Bostrom, and B. Hollback, Geophys. Res. Lett. **21**, 1827 (1994).